Performance Predictions for a Cocurrent Spray Dryer

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A step-by-step prediction procedure has been developed to predict the evaporation of a sodium nitrate spray consisting of small drops moving at terminal velocity through the drying air. The procedure accounts both for the existence of radial gradients as well as radial mixing in the dryer and is selfsustaining except for values of the local air velocity.

Because the procedure can be applied only to the free-fall zone, at some distance below the nozzle, a method was developed for the indirect determination of the amount of water remaining in the partly evaporated droplets at the beginning of the free-fall zone. This procedure was then applied to experimental data on drop size distributions obtained at the beginning of this zone. Owing to uncertainties concernng the salt concentration in the drops, a number of assumptions had to be made and the procedure was only partially successful in describing the spray evaporation, although it did compute the radial spray transfer. It was possible, however, to indicate by means of a model drop distribution the general validity of the procedure.

In this paper, an attempt is outlined to develop a step-by-step calculation procedure for predicting the drying of a spray containing a solute, and the application of this procedure to actual experimental data. The first such procedure was proposed by Marshall (*1*) and it contains many simplifications, the most important of which were rectilinear flow of the air and drops, flat velocity profiles, and the absence of any gradients across the dryer diameter. The present experiments have shown that even in a cylindrical spray dryer with axial air entry these assumptions are questionable. As a result, a more realistic but also a much more complicated model has been devised. In spite of the fact that this model represents an advance, its application is restricted to the region where most of the kinetic energy of the spray jet has been dissipated. The injection of a spray into an air stream is such a violent process, and is initially restricted to such a small fraction of the dryer cross section, that an adequate model does not appear possible, at least at present. The requirements for a treatment of the region of the spray immediately following its generation in the atomizing nozzle would include a knowledge of the rate of entrainment of air by the spray, data concerning the initial drop velocities and directions, and values of the correct drag coefficients for the case of high deceleration and high turbulence. This information is unfortunately lacking for the most part.

In the present study, a step-by-step procedure was developed to predict the progressive evaporation and drying of a spray beyond the so-called "nozzle-range", in this case from the 25 in. level downward. Consequently, the starting data were the experimentally determined parameters at that level which have been already discussed (*2*). From that point on, the calculations made no use of external information, except for the velocity profiles and the radial transfer of air which results from the attenuation of the induced air jet. Because of the complex and repetitious nature of the calculations, the necessary equations were written into a computer program in Fortran IV language and processed on an IBM 7044 computer.

GENERAL CONSIDERATIONS FOR ANALYSIS

Radial Diffusion of Air and Drops

Because of the existence of pronounced gradients in humidity, spray mass velocity and air velocity in the radial direction, it was decided to divide the dryer into a number of concentric zones, consisting of one circular core and nine annuli (for simplicity, the core will be referred to as an annulus). The conditions in each annulus are initially determined from the experimental data at the 25 in. level. These were taken at the radial distance whose circle divided each annulus into two halves. The experiments described in the preceding article (*2*) indicated the existence of considerable radial mixing and in order to be able to estimate the magnitude of this effect the turbulent radial diffusivity was determined. With this parameter it was possible to set up equations to follow the interchange of vapor and drops between adjacent annuli under the assumption that the drops had the same diffusivity as the gas. Soo (*3*) and Chao (*4*) have demonstrated analytically that the turbulent diffusivity of particles too large to follow faithfully the turbulent fluctuations, but small enough to be affected by them, is very nearly equal to that of the gas. This occurs because the reduced turbulent velocity of the particles is offset by the greater persistence of their motion. The diffusivity was assumed to be independent of the radial location.

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Sherwood and Woertz (5) have shown that the diffusivity is indeed constant over the central 80 to 90% of a duct and that it falls to zero at the wall. This was found to be true for fully developed turbulent flow but the nature of the flow in the present case, although turbulent, was unknown.

Although some of the drops were caused to deposit on the dryer walls by the turbulent motion of the air, it was decided to ignore wall deposition in this analysis, both because it was not considered to be very important and because it presented insurmountable analytical problems. Since deposition did not exceed 10% and since most of it occurred in the final stages of drying it cannot be expected to have a marked effect on the drying rate. At any rate, the present state of knowledge does not permit an analytical treatment of the phenomenon, either as regards the rate of deposition of different sized drops or their rate of evaporation following impact.

Interchange between the annuli occurred not only as a result of turbulent mixing but also through the decay of the induced air jet. In order to determine the magnitude of this effect use was made of the velocity profile data. By differentiating the axial variation of air flow in each annulus it was possible to obtain the rate and direction of air flow between annuli as a function of distance from the nozzle. These data were tabulated for use in the computer program.

Drop Transfers

As already stated, the droplets will move laterally between annuli during their downward progress through the drying chamber, although due to the concentration gradient the net movement is away from the dryer axis. Drops of the same original size will evaporate to different extents in different annuli because they have different residence times and are subjected to greater or lesser driving forces. Thus, after one interval, the drops in any one annulus no longer have the same size or water content as the drops in the other annuli. These differences can only become accentuated as the spray descends further. Thus, when the drop interchanges take place the drops which migrate to another annulus cannot be considered to be the same as those already present. In the computation, each class of drops in each annulus is treated separately with regard to this evaporation, so that for the first interval one hundred groups of drops must be considered. It can be easily shown that after every interval the number of groups will increase by a factor of 2.8 and after ten intervals will have grown to three million. Obviously, in these terms the problem becomes intractable both in regard to computer time and computer memory. As the smaller drops become completely dry their respective classes will be dropped from the computation so that the number of groups will increase less rapidly, but it will still tend to astronomic magnitudes.

A solution to this problem is to combine all drops in an annulus, which have come from various sources, into new groups on the basis of the same original diameter and to calculate for each combined group a new weighted diameter and water content. This is done by taking all drops of the same initial diameter in an annulus and by dividing the total water of the group by the total drop number, to arrive at a mean water content and, hence, a mean diameter. Should some of the drops making up a combined group be dry, they will be ignored.

Drop Size Distribution

As it was intended to carry out a mathematical prediction of the spray evaporation, a determination of the drop size distribution was imperative. As already mentioned, it was decided to start the calculations not at the nozzle but at the 25 in. level. Had the prediction been started at the nozzle it would have been necessary to obtain data in its immediate vicinity, a very difficult task, and additional parameters would have had to be investigated. As a result, the drop sizes had to be measured at the 25 in. level, but since drops of different sizes evaporate at different rates the relative salt water content of the drops was some unknown function of their diameter. In other words it was not known how much water the various drops contained nor how the size of a drop would vary with continued water removal during the prediction since its salt content was also unknown.

To overcome this obstacle, the spray at 25 in. below the nozzle was sampled in saturated, room temperature air (cold sample) as well as under normal drying conditions (hot sample). If the results are plotted as number fraction vs. drop size then both plots will have the same area, unity. The cold distribution is then divided into a number of segments by means of vertical lines and the hot distribution is similarly divided into an equal number of segments whose areas are the same as the areas of the corresponding segments in the cold distribution. It can be stated that since the various pairs of segments do have equal areas it follows from the nature of the plots that they contain equal numbers of drops and that, in fact, they contain the same drops in the unevaporated and partly evaporated states, respectively. Thus, by comparing the mean drop sizes of corresponding segments it is possible to estimate the extent of evaporation of each size class and establish a starting point for the prediction.

The procedure outlined above contains some implicit assumptions which need to be examined. It is well known that the atomization of a liquid is affected by the ambient gas density and that a decreasing density leads to the production of coarser drops. Thus the initial drop sizes would be coarser while spraying in hot air. This effect may be expected to be minor for two reasons: (a) The dependence of d_{vs} on the gas density has been shown to be rather weak. Dombrowski and Hooper (6) found that d_{vs} varied inversely as the gas density to the ¼ or 1/6 power; (b) The intense evaporation occurring immediately after the nozzle, where the secondary atomization takes place, will undoubtedly depress the temperature of the air and thereby increase its density.

Perhaps the most important assumption is that it is possible to collect all the drops in both cold and hot conditions, that is, that the samples will be truly representative of the original populations. Should this not be the case the comparison between the distributions will suffer, even though the areas under the curves will always be equal to unity, because some drops will have been excluded or will appear in proportion lower than actual. In either case the proportion of all the other drops in the population will be also incorrect since they are all interdependent. Every known drop size measurement technique breaks down below a certain size and since the small drops are present in large numbers their absence will seriously distort the frequency curve.

These small drops, in spite of their high frequency, represent only a small fraction of the total mass of the spray so that if their presence in the frequency curves could in some way be dispensed with, then their absence would not be felt in the prediction. It may be possible to decide on a cut-off diameter for the cold distribution such that all drops smaller than it can be disregarded but then a corresponding cut-off for the hot distribution must be determined in order that both experimental distributions will contain the same drops and only these. It was observed that drops of sodium nitrate solution behave abnormally at moderate air temperatures, in that the crystals do not form on the drop surface but in the interior. If a drop of

30% solution is assumed to dry completely to a spherical compact mass its diameter will be reduced by 1.83 time. In this way the cut-off diameters of the two distributions can be related to each other. Irrespective of other considerations both these diameters must be large enough that their impaction efficiencies are close to 100%.

The procedure of matching areas under the curves implies that all drops of the same original diameter will have evaporated to the same extent when they reach the 25 in. level. This is not true even for drops following identical paths because atomization is a random process so that not all drops of the same diameter will have the same initial velocity. Moreover, those drops which find themselves at the edges of the spray cone will be in a drier environment and travel through it at a lower velocity than the drops at the axis. These effects reinforce each other so that the former drops will dry more than the latter. In actual fact, the picture is even more complicated in that the spray starts out in the familiar hollow cone pattern, tends towards the center as a result of the partial vacuum created in its core, and then starts spreading out under the influence of turbulent mixing. In addition, the induced air flow which deflects the drops towards the axis entrains primarily the smaller drops which have the lowest inertia. Hence, at the first sampling level the drop size distribution is finer at the axis and coarser towards the edges. Nevertheless, all the drops from the four sampling cells arrayed along a dryer diameter were combined into one gross distribution.

The results from the cold and hot samples appear in Table 1 where under the hot spray heading appear the subheadings of Drops, Crystal Drops, and Particles. The first term needs no explanation, the second indicates that the drops contained one or more salt crystals and under the third are all crystals which did not appear to be associated with any water. The apparent presence of fractional drops is due to the fact that to achieve a properly weighted average the number of drops in the central cell was divided by 3. What is disturbing in Table 1 is the simultaneous presence in the hot spray of drops, crystal drops, and particles, all of the same size. It would have been more according to expection if the three were in distinct size ranges with little overlapping, if any at all. To have this happen in the combined distribution may be attributed to radial variations in dryness, but overlapping occurred even in individual cells. It is unlikely that the small drops resulted from the shattering of large ones because the largest observed drops were well below the critical size for drops impacting on the collection liquid (7). One possible explanation for this overlapping may be that it is caused by drops which move vertically in a more or less rectilinear fashion for the greater part of their trajectories and shortly before capture undergo a large radial displacement as a result of turbulent fluctuations.

The results of the cold to hot drop size distribution comparison are shown in Table 2. Since there were extremely few drops of any kind below 15.1μ in the hot distribution, it was decided to take this as the cutoff diameter. Then the cutoff point for the cold distribution was set at 15.1 × 1.83 = 27.7μ which is the lower limit of class 3. The first two classes of the cold distribution were then discarded and new frequencies were calculated for the remaining ten. As the first two classes contained exactly half the drops, the frequencies for the truncated distribution were simply doubled. The two frequency curves

Table 1. Drop Size Data

		COLD SPRAY		HOT SPRAY				
Class Limits μ	d_i μ	N_i	n'_i %	Drops N_i	Crystal drops N_i	Particles N_i	Total N_i	n'_i %
15.1-20.9	17.5	1,488	29.11	48	390	437	875	24.08
20.9-27.7	24.3	1,068	20.89	337	568	149	1,054	29.00
27.7-34.5	31.1	822	16.09	488	346	10	844	23.20
34.5-41.3	37.9	697	13.64	321	140		461	12.68
41.3-48.1	44.7	475	9.30	235	15.7		251	6.90
48.1-54.9	51.5	278	5.44	81	5.7		87	2.39
54.9-61.7	58.3	136	2.66	41			41	1.13
61.7-68.5	65.1	80.2	1.57	15.7			15.7	0.43
68.5-75.4	72.0	36.9	0.72	7			7	0.19
75.4-82.2	78.8	14.5	0.28	2			2	0.05
82.2-89.0	85.6	11.6	0.23					
89.0-95.8	92.4	4	0.08					
		5,111		1,576	1,455	596	3,627	

Table 2. Drop Size Distribution Comparison

	COLD SPRAY		HOT SPRAY				
Class No.	Limits μ	d_i^* μ	Limits μ	d_i μ	Cold-hot limit ratios	d_i^*/d_i	d_i^3/d_i^{3*}
1	15.1-20.9	17.5					
2	20.9-27.7	24.3					
3	27.7-34.5	31.1	15.1-23.2	18.75	1.83-1.49	1.66	0.218
4	34.5-41.3	37.9	23.2-29.2	26.2	1.49-1.42	1.45	0.328
5	41.3-48.1	44.9	29.2-34.8	32.0	1.42-1.38	1.40	0.364
6	48.1-54.9	51.5	34.8-40.6	37.7	1.38-1.35	1.37	0.389
7	54.9-61.7	58.3	40.6-46.1	43.35	1.35-1.34	1.35	0.407
8	61.7-68.5	65.1	46.1-52.1	49.1	1.34-1.32	1.33	0.425
9	68.5-75.4	72.0	52.1-57.7	54.9	1.32-1.31	1.32	0.435
10	75.4-82.2	78.8	57.7-63.8	60.75	1.31-1.29	1.30	0.454
11	82.2-89.0	85.6	63.8-70.5	67.15	1.29-1.26	1.28	0.476
12	89.0-95.8	92.4	70.5-78.9	74.7	1.26-1.22	1.24	0.524

appear in Figure 1. Starting at the high end, the areas of the successive segments were matched by planimetry. The last column of Table 2 shows the volume ratio between corresponding hot and cold drops. The highest value of this ratio was 0.524. It can be shown from the initial concentration of the drops and the physical properties of sodium nitrate solutions (*8*) that the concentration of an evaporating drop will reach saturation when its volume is reduced to 0.545 of the original. Thus, the volume ratios suggest that all hot drops should contain crystals but this was far from being true.

From the comparison of the distributions it is possible to estimate the extent of evaporation of the spray. It is assumed that the drops smaller than 15.1μ which were not included in the cold sample contain a negligible mass of the spray. By calculating the contribution of each class to the total it is seen that 78.8% of the original water has evaporated. This figure is in very good agreement with 74.9% as derived from the humidity balance for the 25 in. level, in spite of the weaknesses which have been reported.

It should be stated explicitly that the drop size distribution at the 25 in. level was considered to be uniform across the dryer diameter and that all drops of the same original diameter were considered to have evaporated to the same extent regardless of radial position. Because the spray appeared to contain somewhat less water, as a result of the distribution comparison, than indicated by the humidity balance, the water left in the drops was slightly increased (about 4%) in order to make the two agree. The comparison indicates that two of the twelve size classes are completely dry. The additional water was apportioned to the ten classes which were still moist in such a way as to keep the amounts of water evaporated by the various classes in the same proportions as before. The changes in the drop diameters were too small to affect the evaporation calculations to any extent. After the water had been added to the drops there were two dry classes, eight classes whose salt concentration was greater than saturation, and two classes below saturation. The drop diameters with and without evaporation were the ones calculated from the wet bulb temperature of the saturated solution.

Drying Drop Model

Drying droplets of sodium nitrate solution crystallize in a peculiar manner at the moderate temperature used in the present experiments. No crust is formed on the drop surface but one or more crystals form in the drop interior. As drying proceeds, the drop shrinks and the crystal mass in its interior grows outward. In the majority of cases this mass is composed of many small crystals and when the liquid surface recedes into the interior of the structure the latter disintegrates, probably as a result of vapor evolution in the interstices. Such a random behavior can not be modeled mathematically. It was assumed that no concentration gradients existed in the drops and that the precipitated salt deposited on the surface of a spherical salt mass at the drop center (sodium nitrate crystallizes in the cubic system and in fact some drops did produce single cubical crystals). It was further assumed that the drop continued to dry until all the water had evaporated and all the salt had gone to build up the hypothetical sphere. The inability to account for the disintegration of the drops will, most probably, lead to an underestimation of the drying rates, as it may be safely assumed that upon disintegration the remaining water will evaporate at a suddenly enhanced rate. It was felt, however, that disintegrating drops contained little water so that their contribution to the overall rate should not be appreciable. It was assumed that the surface concentration of all the drops had attained saturation. Only two classes had bulk concentrations below saturation and their surface concentrations could not be calculated because this is still an unresolved problem. The saturation concentration was that prevailing at the wet bulb temperature. The latter was obtained from data in the Landolt-Börnstein tables (*8*) but was corrected for the heat of solution of sodium nitrate as given by Mondain-Monval (*9*). The correction was a small one.

Time Increments

In a step-by-step calculation all the rate equations are applied for a small interval of time. At the end of this interval the drops are slightly dryer and have descended to a lower level. The new level is simply calculated from the air velocity and the time interval. In the present case, however, the air velocity in each annulus was different so that if the same time interval were used for each annulus, the central annulus would outdistance the others. Consequently, it would be impossible to carry out the calculations for interchange of humidity and droplets between annuli, since this annulus would be surrounded by a region of unknown properties. It thus became necessary to use different time intervals for all annuli, so that they would advance equally. The time intervals chosen for the annuli were inversely proportional to their velocities.

It was assumed that the drops will travel down the dryer at the same rate as the local air velocities. In actual fact the larger drops do have relative velocities which are of the order of several percent of the air velocity so that their residence times will be shorter than computed. On the other hand, the relative velocity between air and drops will result in increased evaporation rates so that the two effects tend to cancel each other out. It can be shown that the net difference is of the order of a few percent, at worst. In view of the other assumptions in the prediction procedure the effects of the relative velocity have been ignored especially since a more rigorous treat-

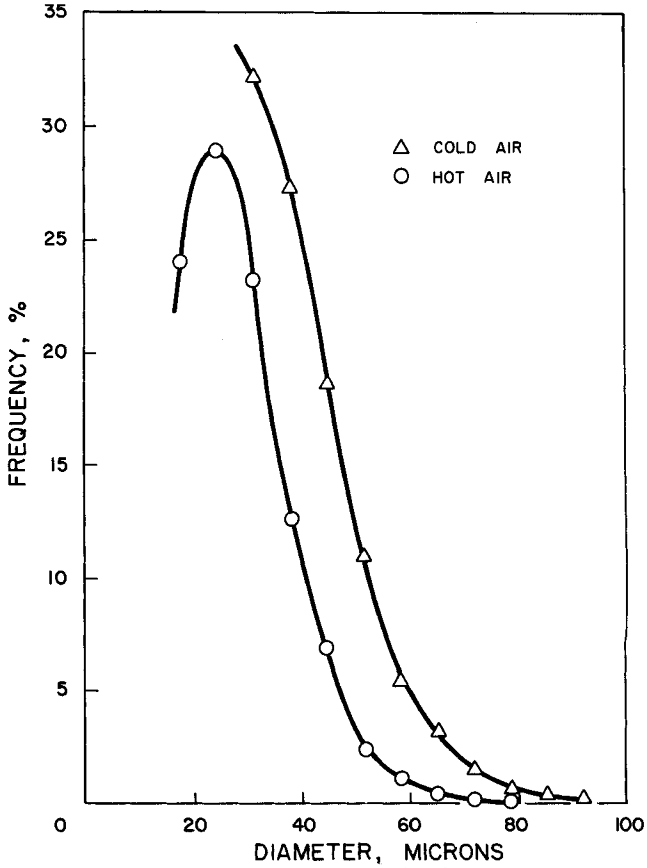

Fig. 1. Cold and hot drop size distributions.

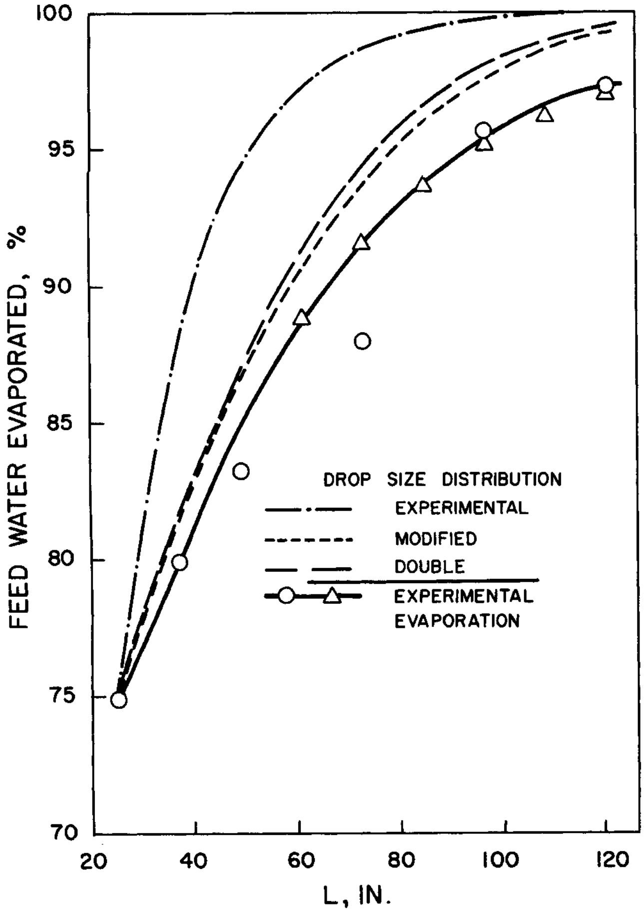

Fig. 2. Predicted and experimental evaporation.

ment would have been quite complicated.

DEVELOPMENT OF WORKING EQUATIONS

It was first necessary to develop equations to calculate the local air properties. The air temperature, air density, and water vapor diffusivity are given by Equations (1), (2) and (3) respectively:

$$T_c = T_{co} + 33.2 - 4170\,H \qquad (1)$$

$$\rho = 39.8/(460 + T_c) \qquad (2)$$

$$D_v = (2.043 \times 10^{-7})(920 + T_c + T_{wb})^{1.75} \qquad (3)$$

Each step of a step-by-step calculation procedure for the prediction of spray evaporation consists of many individual calculations, the first of these being the termination of the time interval for each annulus. For a typical annulus, j, the time interval $\Delta\theta_j$ as a function of the centerline interval $\Delta\theta$ is

$$\Delta\theta_j = \Delta\theta(V_{\text{℄}}/V_j) \qquad (4)$$

The lateral transfer area between annuli j and $j + 1$ is:

$$A_j' = 0.294V\Delta\theta/\log\,[(R_j + 0.33)/R_j] \qquad (5)$$

iThe change in the water content l_k of a drop of diameter d_k is calculated by

$$\Delta l_k = 1.431 \times 10^{-7}\,D_v\rho\Delta\theta_j(H_{wb} - H_j) \qquad (6)$$

Equation (6) is repeated for every drop size in the annulus. Then the new water content is simply found by substracting Δl_k from the original l_k. Should the result be negative it is replaced by zero and Δl_k is made equal to the original l_k.

The water evaporated by drops of one size class in an annulus is

$$w_k = N'_k\Delta l_k \qquad (7)$$

where N'_k is the number of drops of class k in the annulus. The total evaporation w_j in an annulus is obtained simply by summing the w_k terms.

The diameter of a drop after evaporation is obtained as a function of the ratio of its water content to its original water content and of the original diameter. There are two distinct equations for drops below and above saturation. The humidity in each annulus at the end of the interval is obtained by taking into account the amount of water evaporated and the vapor transfer between the annuli. The properties at the end of the interval are denoted by the subscript 2. For the j annulus:

$$\begin{aligned} H_{j,2} = {} & [A_jV_j\rho_jH_j + D_tA'_{j-1}(H_{j-1}\,\rho_{j-1}\rho_j)/0.0333 \\ & - (W_{j-1}H_{j-1}V_L\Delta\theta/3600 + w_j\,/3600 - D_tA_j'(H_j\rho_j \\ & - H_{j+1}\rho_{j+1})/0.033 + W_jH_jV_{\text{℄}}\Delta\theta/3600]\,/\,(A_jV_{j,2}\rho_{j,2}) \qquad (10) \end{aligned}$$

The denominator of the above equation contains the term $\rho_{j,2}$ which depends on $H_{j,2}$, the result of the calculation and consequently, two iterations are performed.

The drop transfer calculations are applied to all the drops irrespective of size. There are four separate expressions for drops moving away from or toward the axis under the influence of either turbulent mixing or the expansion of the jet. When these equations have been solved it is possible to calculate the number of drops which stayed in the annulus throughout the interval, the total number of drops at the end of the interval and the fractional mass of dry salt per annulus.

Following the drop interchanges, the drops can be combined into new classes and the mean water content and diameter for the drops of each class can be found. The transfer equations operate on all drops regardless of the fact that they may have become dry salt particles. It is possible to have in an annulus both drops and particles derived from drops of the same original size but as already stated the latter are not included in the averaging. It is therefore convenient to define for each group of drops and particles a parameter which is equal to the number

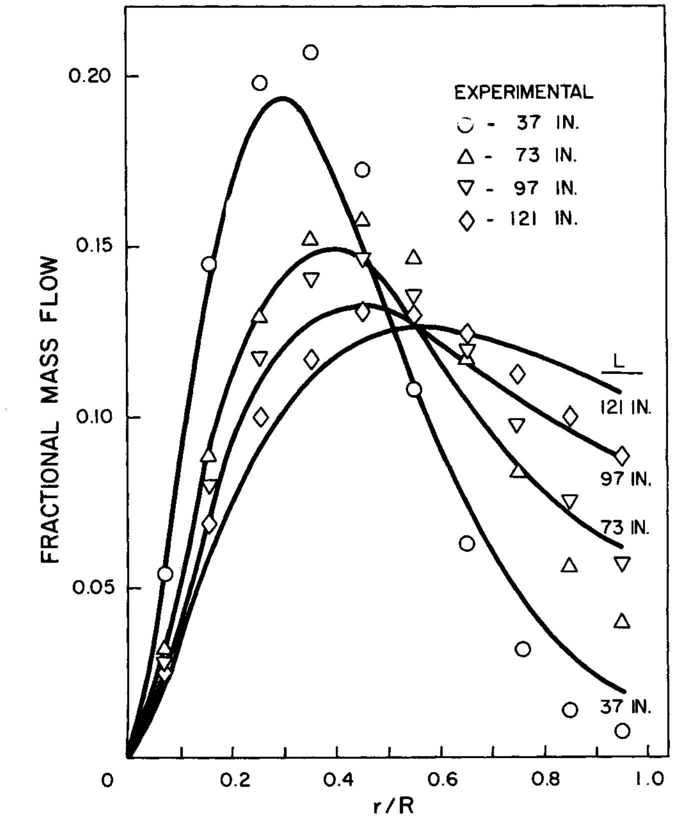

Fig. 3. Radial salt mass flow profiles.

of wet drops in that group divided by the class frequency. Then for each class which has any wet drops at the end of the interval the mean water content can be calculated and this is also the water content for the start of the next interval. From the new water content it is then possible to obtain the new diameter for each size class and by combining all the water contents the spray moisture in each annulus is easily obtained.

With the calculation of the spray moisture, one interval has been completed. All the rate equations, as well as those that make use of their results, are strictly true over an infinitesimal time increment at the start of each step, because the concentration differences on which they are based will have decreased at the end of the step. Thus the procedure outlined above overestimates the various effects. A correction may be applied by repeating the calculations for the conditions prevailing at the end of the step and then averaging the two sets of results, but this would have further increased the complexity of the computer programming. Since the errors inherent in using finite time increments can be expected to increase through the use of larger increments, $\Delta\theta$ values of 0.001 and 0.007 sec. were used. The differences produced through the use of the larger increment were negligible so it was assumed that with $\Delta\theta = 0.001$ sec. no correction was required.

DISCUSSION

The single most important result from the plethora of parameters turned out by the computer is the fraction of feed water evaporated as a function of axial distance. It is obvious from Figure 2 that the predicted evaporation proceeds much more rapidly than the experimental one, especially at the start of the computation where the predicted evaporation rate was 3.5 times greater than actual. Such a high rate is due to the assumed presence of a large number of small drops, because small drops evaporate much more rapidly than large ones. Consequently, the drop size comparison failed, in that it made it appear that many small drops were still wet while, in reality, they had become dry. A number of assumptions were made in developing the comparison technique, and these are reviewed concisely:

1. The impaction efficiencies for all the drops are equal to unity.
2. The drops settle vertically in the collection cells.
3. The local distributions may be combined into one overall distribution.
4. All portions of the spray at a dryer level have the same moisture content, irrespective of radial location.
5. Sufficient drops were sized in order to characterize the distribution.
6. The difference in atomization conditions for the two drop size samplings, in hot and cold air, was negligible.
7. The cutoff diameters of the two distributions were in the proper relation to each other.

The computations have demonstrated assumption (4) to be quite weak. At the 25 in. level, a drop near the wall will lose 6.2 times as much water by evaporation as an identical drop near the axis for the same vertical descent. This results from the lower driving force and lower residence time prevailing at the axis. Starting with the assumed uniform spray moisture of 0.585 lb. water/lb. salt at 25 in. from the nozzle, the computation showed that at 36 in. the moisture at the axis has fallen to 0.419 while at the wall it is already down to 0.105 lb. water/lb. salt. From these figures it may be inferred that a considerable spray moisture gradient must also exist at the 25 in. level.

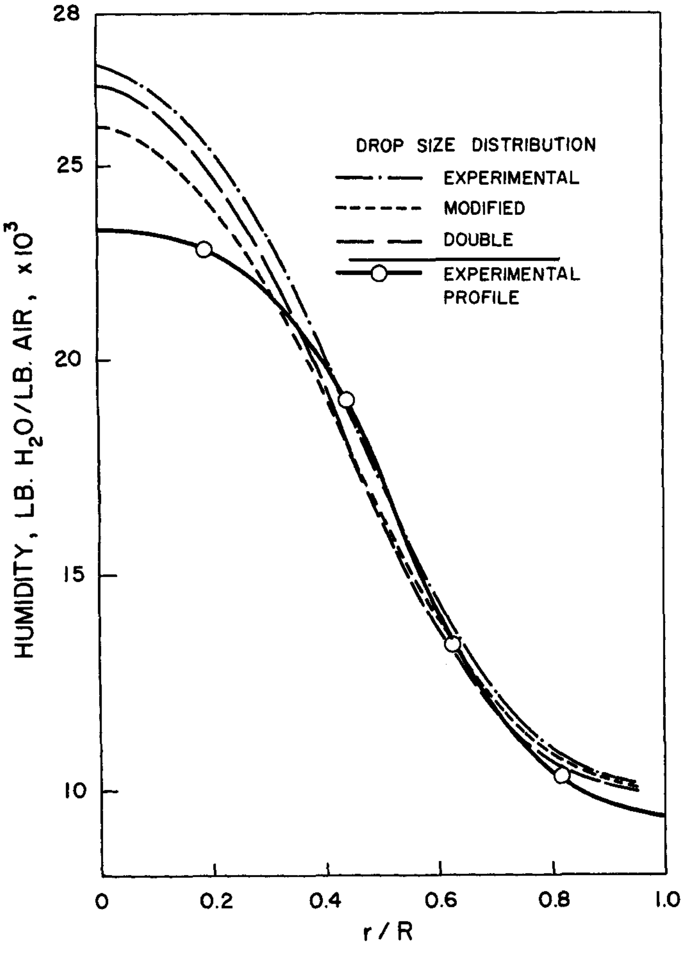

Fig. 4. Radial humidity profiles 49 in. level.

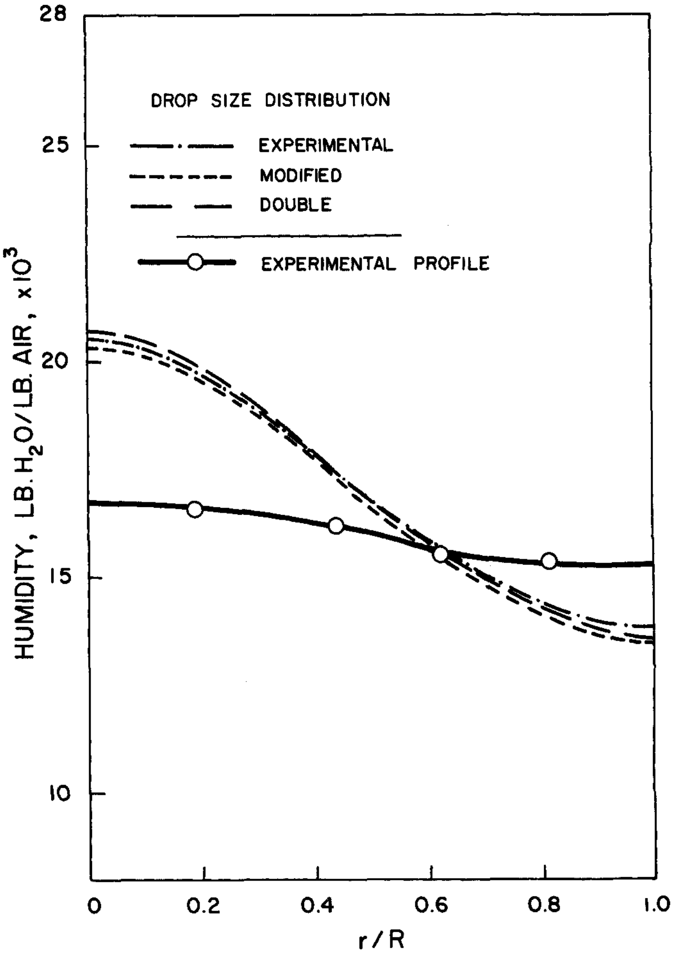

Fig. 5. Radial humidity profiles 121 in. level.

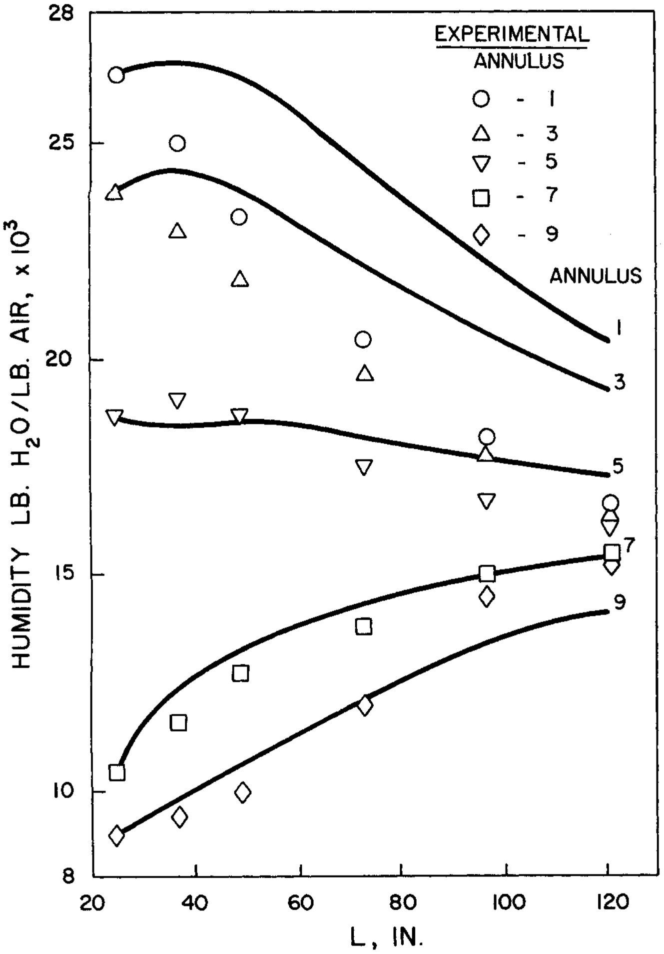

Fig. 6. Axial humidity profiles predicted from experimental hot distribution.

Other results of the computer calculations included the local humidity and the local mass flow of dry salt. A comparison of computed and experimental profiles of the latter appears in Figure 3. With the exception of the region near the wall the agreement is very satisfactory so that the validity of the assumption regarding the equality of the turbulent diffusivities of air and drops is confirmed.

The agreement in the case of the humidity profiles was considerably less satisfactory (Figures 4 to 6). This can be traced to the fact that the local humidity depends not only on the turbulent diffusivity but also on the local rate of vapor evolution which, as has been seen, was higher than the experimental. Figures 4 and 5 show that the discrepancy both at the axis and the wall becomes greater with increasing distance from the nozzle, but is small for intermediate radial positions.

The evaporation prediction computation was repeated with a new hot distribution which resulted in an evaporation rate more closely in agreement with the experimental. This distribution was termed the *modified* distribution and the resultant evaporation is shown in Figure 2. The starting point for the modified distribution was the experimental cold distribution. The criteria were that a suitable distribution would have no small wet drops but would still contain the appropriate amount of water. Consequently, the larger drops would have to contain more water than indicated by the comparison. The modified distribution was obtained from the cold distribution by a process of trial and error and the water assigned to the wet drop classes determined simply by having the ratios of present water to original water fall along an arbitrary, smooth curve going from zero to a value less than unity. Several such distributions were used in the computation until one was found that gave satisfactory results. This distribution is compared to the hot distribution in Figure 7. The appropriate humidity profiles appear in Figures 4 and 5. It is apparent that the modified and the experimental hot distributions are quite different and that although the computed humidity profiles are somewhat closer to the experimental ones, there still remain marked differences especially in the vicinity of the dryer axis.

Because in reality the drop size distribution was not uniform across the dryer, tending to be coarser away from the axis, and because, as has been mentioned, there is good reason to suspect that there were considerable spray moisture gradients at the starting point of the prediction, it was decided to take these two facts into account in the prediction, if only in part. Since the first three annuli contained nearly half the salt (45.2%) at the 25 in. level, it was decided to divide the dryer cross section into two portions, inner and outer, with the inner coinciding with the first three annuli. Each portion would have, at the start of the prediction, its own drop size spectrum and its own moisture content. The inner portion contained only 37.7% of the evolved vapor and with the assumption that the diffusivities of drops and vapor are identical in the nozzle zone, the spray moisture in the inner and outer portions was 0.873 and 0.345 lb water/lb. salt, respectively. In order to obtain the appropriate hot distributions, the cold distribution was divided into two distributions, the finer one for the inner portion and the coarser for the outer portion. The available water in either portion was distributed among the drops in the same way as for the modified distribution. The two distributions were termed the *double* distribution. For the purpose of comparison the inner and outer distributions were combined into one and the resultant distribution appears on Figure 7.

In the computation, the drops of the two distributions

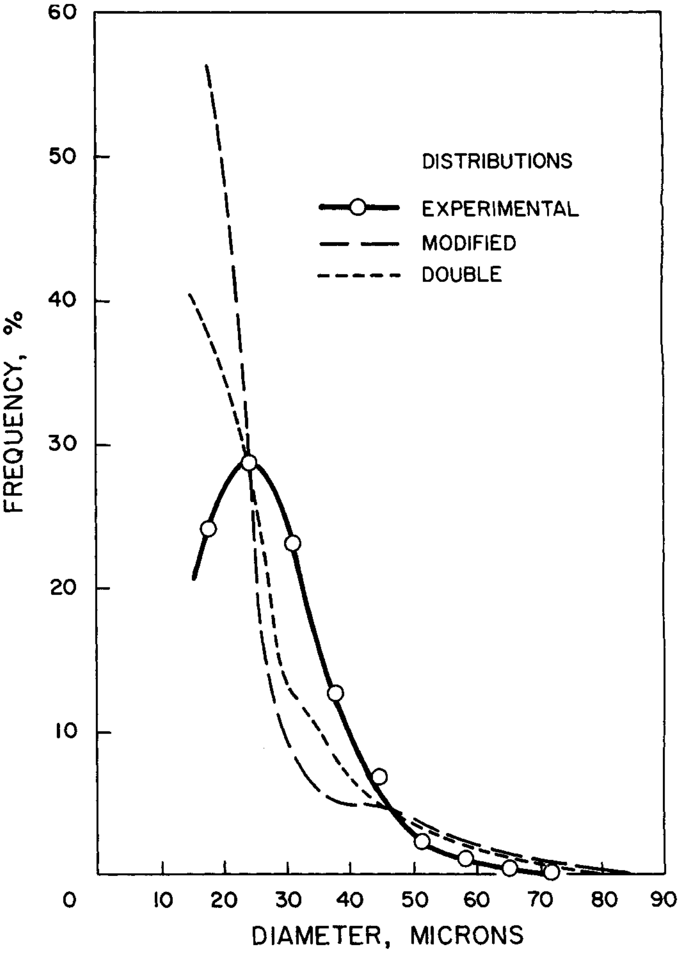

Fig. 7. Comparison of drop size distributions.

were treated independently although, naturally, the total evaporation and the air humidity depended on the contributions for both sets. Although the sets started out confined in separate portions of the dryer cross section, their drops intermingled as a result of turbulent diffusion, so that very soon both sets could be found in all annuli. The evaporation from the double distribution differs very little from that of the modified (Figure 2). The humidity profiles (Figures 4 and 5) also differ little although in the inner annuli the humidity is slightly higher. Although the size spectrum of the double distribution still differs appreciably from that of the experimental hot distribution (Figure 7) it does resemble it more than does the modified. Thus, it may be stated that the above approach is a step in the right direction and corresponds to reality by allowing for the presence of small wet drops. It is by restricting these drops, at least initially, to the portion of the dryer cross section where the evaporation rate is lowest that the overall evaporation rate was also kept in check.

CONCLUSIONS

The step-by-step prediction technique for the evaporation of a spray in the free-fall zone of a dryer was intended to account for the flow phenomena described in the preceding article (2) and is much more ambitious than any previous published attempts in this direction. The technique appears to be sound both because it successfully predicted the radial spreading of the spray and because the various trends are in the proper direction. It has been only partially successful in predicting the evaporation of the spray from the experimental data because of the impossibility of establishing, by means of the measurement techniques used, the precise quantities of water remaining in the various classes of droplet sizes of the partly dry spray at the starting level (25 in. from the nozzle). The large number of assumptions involved in this procedure probably militated against the correct apportionment of the water, remaining at the 25 in. level, among the various size classes. Nevertheless, it has been possible to indicate fairly conclusively that the most objectionable of these assumptions may be dispensed with by obtaining a number of local drop size spectra corresponding to the various radial locations, for both the unevaporated and the partly evaporated spray.

It is reasonable to suppose that in an industrial spray dryer the starting water contents may be more easily established because the higher spray/air loading and the much larger spray flow rate should lead to an appreciably reduced evaporation in the nozzle zone. Consequently, the comparison of the hot to cold size distributions should be simpler and produce better results. In the present case it was necessary to make the comparison and to carry out the prediction calculations with only one-quarter of the feed water still in the liquid state.

As the radial diffusion of the spray seems to have been successfully predicted on the basis of equal turbulent diffusivities for the spray and the gaseous phase it can be concluded that the correct value of the diffusivity was used in the computation. It then follows that discrepancies between experimental and predicted humidity profiles are due to the incorrect rates of local evaporation.

The computations have shown unequivocally that the assumptions of plug flow and the absence of radial gradients, often employed in previous prediction techniques, are without foundation. It can be expected that these assumptions will have a greater effect on the life time of the largest droplet than on the overall spray evaporation. As the ultimate aim of any evaporation prediction should be the determination of the above lifetime for the purpose of equipment sizing, these simplifying assumptions will lead to serious errors.

Even though the computations were fairly complex, they apply only to a very simple system. A prediction procedure for an industrial dryer would have to account for such additional complications as multiple spray nozzles, multiple air inlets, swirling flow, dryer diameter variations and counter-current spray injection. With sufficient knowledge of the flow conditions, no serious difficulty in the programming of these additional factors is anticipated.

NOTATION

A = cross sectional area of annulus, sq.ft.
A' = annular transfer area, sq.ft.
d = drop diameter, μ
d_{vs} = volume-surface mean diameter, μ
D_t = turbulent diffusivity, sq.ft./sec.
D_V = diffusivity of water vapor, sq.in./sec.
H = humidity, lb. water/lb. air
H_{wb} = saturation humidity at wet bulb temperature, lb. water/lb. air
l = mass of water in drop, lb.
n' = drop frequency
N = rate of drops at start of interval, drops/hr.
N_i = number of drops in ith class of sample
N' = rate of drops of given class at start of interval, drops/hr.
R = outside radius of annulus, ft.
T_c = air temperature, °F.
T_{co} = initial air temperature, °F.
T_{wb} = wet bulb temperature, °F.
v_i = volume fraction of ith class
V = air velocity, ft./sec.
$V_{℄}$ = air velocity at axis, ft./sec.
w = evaporation rate, lb./hr.
W = air transfer rate due to jet expansion, lb./hr.ft.
X = spray moisture, lb. water/lb. salt
θ = time increment (interval), sec.
ρ = air density, lb./cu.ft.

Subscripts

j = annulus
k = drop size class
2 = end of interval

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